NUMBER SYSTEMS

Numbers are intellectual witnesses that belong only to mankind.

1. If the H C F of 657 and 963 is expressible in the form of 657x + 963**x** - 15 find x. (Ans:x=22)

Ans: Using Euclid's Division Lemma

$a = bq + r$, $o \le r < b$

$963 = 657 \times 1 + 306$

$657 = 306 \times 2 + 45$

$306 = 45 \times 6 + 36$

$45 = 36 \times 1 + 9$

$36 = 9 \times 4 + 0$

$\therefore$ HCF (657, 963) = 9

now $9 = 657x + 963 \times (-15)$

$657x = 9 + 963 \times 15$

$= 9 + 14445$

$657x = 14454$

$x = 14454/657$

$$\boxed{x = 22}$$

2. Express the GCD of 48 and 18 as a linear combination. (Ans: Not unique)

A=bq+r, where $o \le r < b$

48=18x2+12

18=12x1+6

12=6x2+0

$\therefore$ HCF (18,48) = 6

now 6= 18-12x1

6= 18-(48-18x2)

6= 18-48x1+18x2

6= 18x3-48x1

6= 18x3+48x(-1)

i.e. 6= 18x +48y

$\therefore$ $\boxed{x=3 , y=-1}$

$6 = 18\times 3 + 48\times(-1)$
$= 18\times 3 + 48\times(-1) + 18\times 48 - 18\times 48$
$= 18(3+48) + 48(-1-18)$
$= 18\times 51 + 48\times(-19)$
$6 = 18x + 48y$

$\therefore$ $\boxed{x = 51,\ y = -19}$

Hence, x and y are not unique.

3. Prove that one of every three consecutive integers is divisible by 3.

Ans:

n,n+1,n+2 be three consecutive positive integers
We know that n is of the form 3q, 3q +1, 3q + 2
So we have the following cases

Case – I when n = 3q

In the this case, n is divisible by 3 but n + 1 and n + 2 are not divisible by 3

Case - II When n = 3q + 1
Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

Case – III When *n* = 3q +2
Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

Hence one of n, n + 1 and n + 2 is divisible by 3

4. Find the largest possible positive integer that will divide 398, 436, and 542 leaving remainder 7, 11, 15 respectively.
(Ans: 17)

Ans: The required number is the HCF of the numbers

Find the HCF of 391, 425 and 527 by Euclid's algorithm

$\therefore$ HCF (425, 391) = 17

Now we have to find the HCF of 17 and 527
527 = 17 x 31 +0

$\therefore$ HCF (17,527) = 17

$\therefore$ HCF (391, 425 and 527) = 17

5. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

(Ans:2520)

Ans: The required number is the LCM of 1,2,3,4,5,6,7,8,9,10

$\therefore$ LCM = $2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 2520$

6. Show that 571 is a prime number.

Ans: Let $x=571 \Rightarrow \sqrt{x}=\sqrt{571}$

Now 571 lies between the perfect squares of $(23)^2$ and $(24)^2$
Prime numbers less than 24 are 2,3,5,7,11,13,17,19,23
Since 571 is not divisible by any of the above numbers
571 is a prime number

7. If d is the HCF of 30, 72, find the value of x & y satisfying d = 30x + 72y.

(Ans:5, -2 (Not unique)

Ans: Using Euclid's algorithm, the HCF (30, 72)

$72 = 30 \times 2 + 12$
$30 = 12 \times 2 + 6$
$12 = 6 \times 2 + 0$

HCF (30,72) = 6
$6=30-12\times2$
$6=30-(72-30\times2)2$
$6=30-2\times72+30\times4$
$6=30\times5+72\times-2$
$\therefore$ x = 5, y = -2

Also $6 = 30 \times 5 + 72 (-2) + 30 \times 72 - 30 \times 72$

Solve it, to get

$x = 77, y = -32$

Hence, x and y are not unique

8. Show that the product of 3 consecutive positive integers is divisible by 6.

Ans: Proceed as in question sum no. 3

9. Show that for odd positive integer to be a perfect square, it should be of the form 8k +1.

Let a=2m+1

Ans: Squaring both sides we get

$a^2 = 4m (m +1) + 1$

$\therefore$ product of two consecutive numbers is always even

m(m+1)=2k

a^2=4(2k)+1

$a^2 = 8 k + 1$

Hence proved

10. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36. (Ans:999720)

Ans: LCM of 24, 15, 36

LCM = $3 \times 2 \times 2 \times 2 \times 3 \times 5 = 360$

Now, the greatest six digit number is 999999

Divide 999999 by 360

$\therefore$ Q = 2777 , R = 279

$\therefore$ the required number = 999999 – 279 = 999720

11. If a and b are positive integers. Show that √2 always lies between $\frac{a}{b}$ *and* $\frac{a-2b}{a+b}$

Ans: We do not know whether $\frac{a^2-2b^2}{b(a+b)}$ or $\frac{a}{b} < \frac{a+2b}{a+b}$

$\therefore$ to compare these two number,

Let us comute $\frac{a}{b} - \frac{a+2b}{a+b}$

=> on simplifying , we get $\frac{a^2-2b^2}{b(a+b)}$

$\therefore\ \frac{a}{b} - \frac{a+2b}{a+b} > 0$ or $\frac{a}{b} - \frac{a+2b}{a+b} < 0$

now $\frac{a}{b} - \frac{a+2b}{a+b} > 0$

$\frac{a^2 - 2b^2}{b(a+b)} > 0$ solve it , we get , $a > \sqrt{2}b$

Thus , when $a > \sqrt{2}b$ and

$\frac{a}{b} < \frac{a+2b}{a+b}$,

We have to prove that $\frac{a+2b}{a+b} < \sqrt{2} < \frac{a}{b}$

Now $a > \sqrt{2}\, b \Rightarrow 2a^2+2b^2 > 2b^2 + a^2 + 2b^2$

On simplifying we get

$\sqrt{2} > \frac{a+2b}{a+b}$

Also $a > \sqrt{2}$

$\Rightarrow \frac{a}{b} > \sqrt{2}$

Similarly we get $\sqrt{2}, < \frac{a+2b}{a+b}$

Hence $\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$

12. Prove that $(\sqrt{n-1} + \sqrt{n+1})$ is irrational, for every $n \in N$

Self Practice